Variability of wind speed encountered by a vessel

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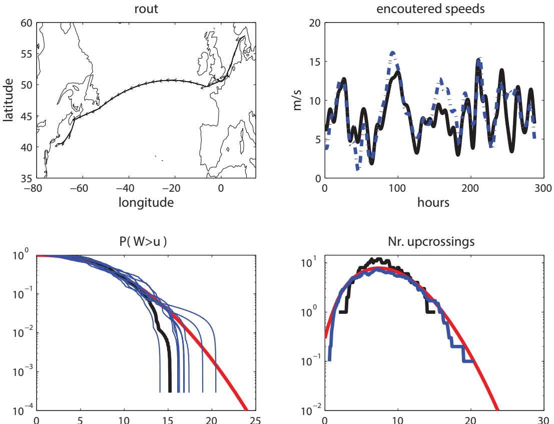
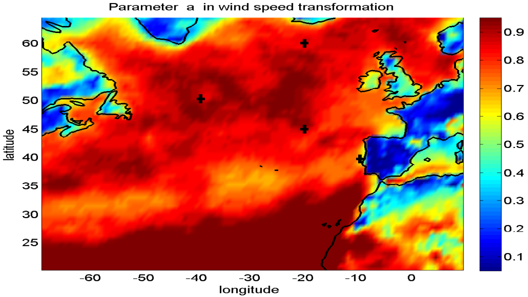


Wind speed $W(\mathbf{p}, t)$ model

Brown et al. (1984) $W(\mathbf{p}, t)^{a(\mathbf{p})} = X(\mathbf{p}, t)$.

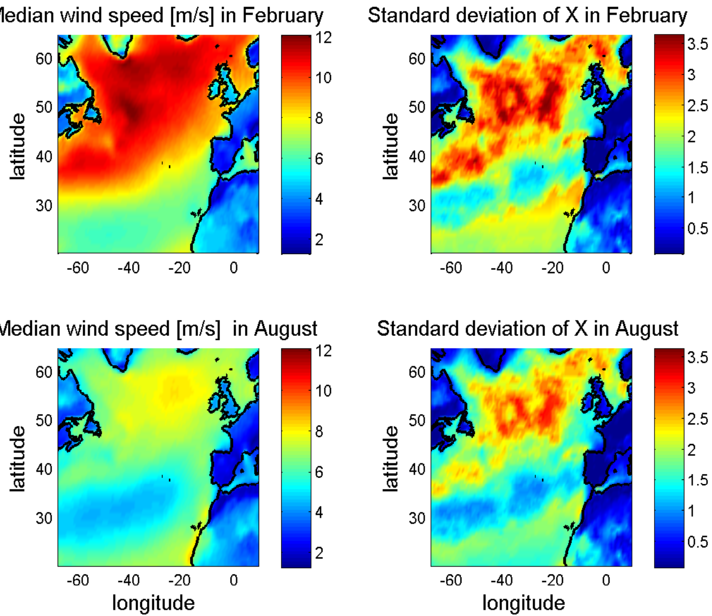
Assumption: X is locally stationary, for couple of weeks and in radius of few degrees.



Application for computations of:

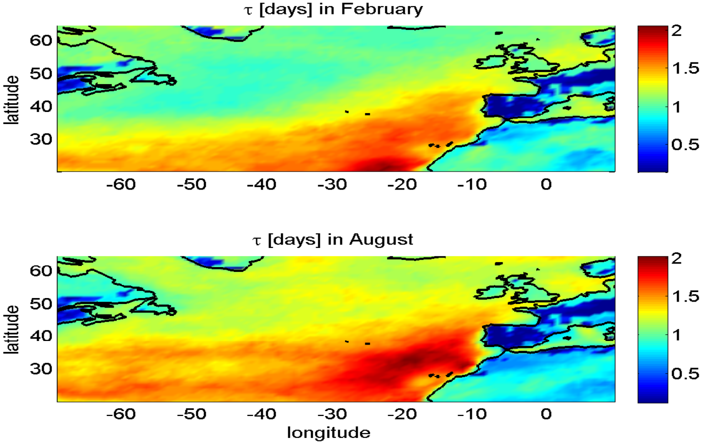
- long-term cdf of encountered wind speeds;
- strength and duration of encountered storms;
- simulation of encountered winds.

Parameters $\mu = E[X(\mathbf{p}, t)]^{1/a(\mathbf{p})}$, $\sigma = \mathbb{V}ar(X(\mathbf{p}, t))^{1/2}$



Duration of windy weather at fixed position and time, i.e. average time between upcrossing and the following downcrossing of the median wind -

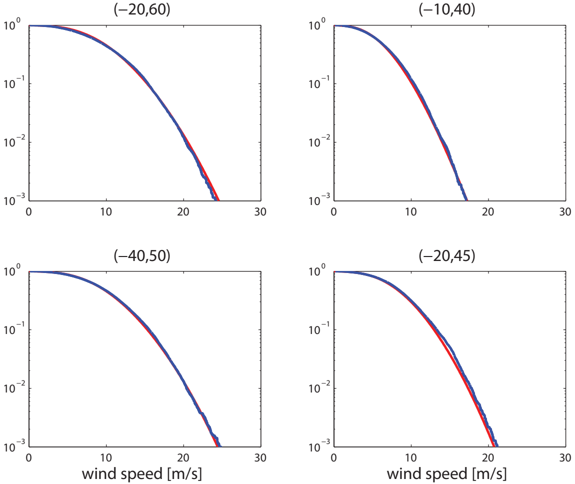
$$\tau(\mathbf{p}, t) = \pi \sqrt{\frac{\mathbb{V}ar(X(\mathbf{p}, t))}{\mathbb{V}ar(X_t(\mathbf{p}, t))}}$$



Long term cdf at fixed position $\mathbf{p}$, T one year,

$$P(W \leq w) = \frac{1}{T} \int_{t}^{t+T} \Phi \left(\frac{w^{a(\mathbf{p})} - m(\mathbf{p}, s)}{\sigma(\mathbf{p}, s)}, \right) \, ds$$

$m(\mathbf{p}, s) = E[X(\mathbf{p}, s)]$



Other wind characteristics at fixed location $\mathbf{p}$, e.g. buoy

- $\{s \in [t, t+T] : W(\mathbf{p}, s) \geq u\}$ - storms (safety),
- t_i, T_i^{st}, A_i^{st} - time when i'th storm starts; its duration and height.
- $P(A^{st} > w) = \frac{E[\#\{A_i^{st} > w\}]}{E[N_T(u)]} \leq \frac{E[N_T(w)]}{E[N_T(u)]}$ [1]
- $P(T^{st} > t) = \frac{E[\#\{T_i^{st} > t\}]}{E[N_T(u)]}$ (WAFO toolbox), $\quad E[T^{st}] = \frac{P(W > u)}{E[N_T(u)]}$

[1] $E[N_T(u)] = \int_t^{t+T} \frac{1}{2\,\tau(\mathbf{p},s)}\, e^{-\frac{(u^{a(\mathbf{p})} - m(\mathbf{p},s))^2}{2\sigma^2(\mathbf{p},s)}}\, ds$

position	$u = 15$ m/s				$u = 18$ m/s			
	$E[T^{st}]$	$\bar{T}^{st}$	$E[T^{cl}]$	$\bar{T}^{cl}$	$E[T^{st}]$	$\bar{T}^{st}$	$E[T^{cl}]$	$\bar{T}^{cl}$
(-20,60)	0.6	0.5	4.4	4.2	0.5	0.4	13.	11
(-10,40)	0.3	0.4	56	69	0.3	0.3	514	525
(-40,50)	0.6	0.5	4.4	4.2	0.5	0.4	12	11
(-20,45)	0.6	0.5	11	13	0.4	0.4	46	57

Table : Long term (one year) expected storm/calm durations in days.

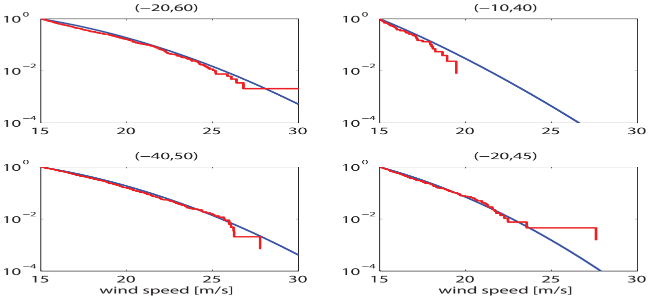


Figure : Comparisons of estimates of long-term probability that wind in a storm exceeds u, $\mathbb{P}(A^{st} > w)$, $u = 15$ [m/s], during one year

Connecting time and space

Σ - covariance matrix of gradien, 6 parameters at each location. These have following physical interpretations.

- θ - main direction of propagation, rotation making (X_x, X_y) uncorr, Σ_θ - covariance matrix in the rotated coordinate system. by angle θ.
- $v_\theta = -\frac{\mathbb{C}ov(X_t^\theta, X_x^\theta)}{\mathbb{V}ar(X_x^\theta)}$ - average speed in direction θ.
- $v_{\theta-90^o} = -\frac{\mathbb{C}ov(X_t^\theta, X_y^\theta)}{\mathbb{V}ar(X_y^\theta)}$ - average speed in direction $\theta - 90^o$,
- $L_\theta = \pi\sqrt{\frac{\mathbb{V}ar(X)}{\mathbb{V}ar(X_x^\theta)}}$ average length of windy region in direction θ
- $L_{\theta-90^o} = \pi\sqrt{\frac{\mathbb{V}ar(X)}{\mathbb{V}ar(X_y^\theta)}}$ average length in direction $\theta - 90^o$
- $\tau = \pi\sqrt{\frac{\mathbb{V}ar(X)}{\mathbb{V}ar(X_t)}}$ average period of the windy weather

Average velocity of storms movement

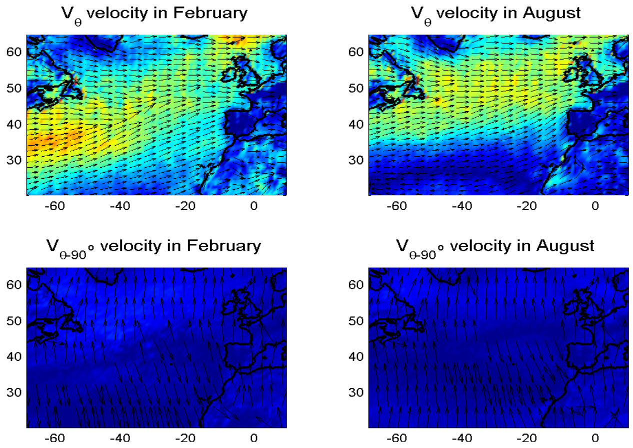
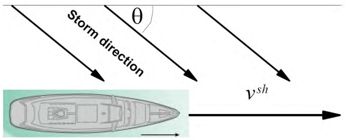


Figure : In the figures max speed is about 44 km/h and minimum 0 km/h

Storm velocity in rotated coordinate system $\mathbf{v}^{st} = (v_\theta, v_{\theta-90^o})$.

Average period of sailing in windy weather τ^e



- A rout defines by speed $v^{sh}(s)$, azimuth $\alpha(s)$. Then encounter wind velocity in the rotated, by θ, coordinates

$$(v^e_\theta(s), v^e_{\theta-90^o}(s)) = \mathbf{v}^{st}(s) - \mathbf{v}^{sh}(s),$$

$$\mathbf{v}^{sh}(s) = v^{sh}(s) \cdot (\cos(\alpha(s) - \theta(s)), \ \sin(\alpha(s) - \theta(s))).$$

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$$\tau^e(s) = \frac{1}{\sqrt{(v^e_\theta/L_\theta)^2 + (v^e_{\theta-90^o}/L_{\theta-90^o})^2 + (1/\tau)^2 \cdot (1 - \alpha^2_\theta - \alpha^2_{\theta-90^o})}}.$$

Simulation

Common experience says that wind speeds varies in different time scales, e.g. diurnal patten due to different temperatures at day and night; frequency of depressions and anti-cyclones which usually occur with periods of about 4 days and annual pattern.

To follow the claim $x(\mathbf{p}, s)$ were decomposed (fft) into four parts: containing periods above 40 days, between 40 and 5 days, between 5 and 1 day and noise. For each signal the parameters have been estimated. Then the Gaussian process $X(s)$ or $X^e(s)$ is simulated by

$$X(s) = m(s) + \sum_{i=1}^{4} \int_{-\infty}^{+\infty} \sigma_i(s) f_{\tau_i(s)}(s-t)\, dB_i(t),$$

where

$$f_\tau(t) = (2/\pi)^{1/4} \frac{1}{\sqrt{\tau}} \exp\left(-\pi^2 \left(\frac{t}{\tau}\right)^2\right).$$

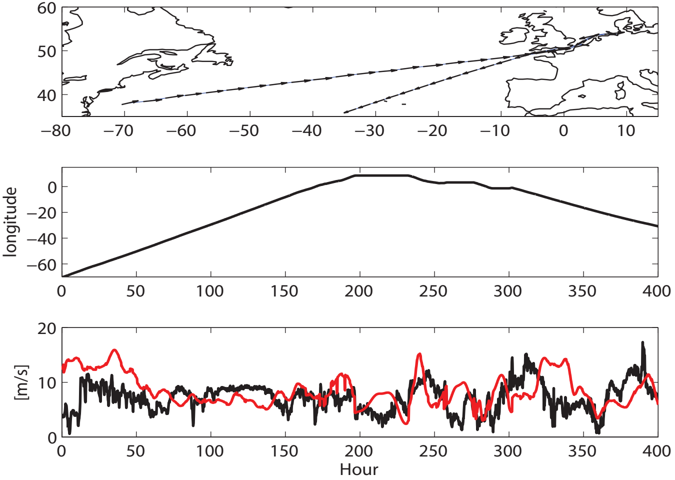

Figure : *Top* - A rout sailed in Northern Atlantic in April. *Middle* - The longitude as a function of sailing time [hours]. *Bottom* - Wind speeds measured on-board a vessel (black line) and a simulation of the wind speed by means of the model (red line).

THANK YOU FOR ATTENTION