Modeling and Infering a Spatio-temporal Dynamic For Apple Scab in Orchards

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motivation

Goal:
- To describe and statistically infer epidemiologic parameters of apple scab
- To answer the question: How much mixture plantation affect scab dynamics?

Experimental essay
- 9 contiguous apple orchards of 2 types
  - Pure: only susceptible cultivars (melrouge variety): 3 orchards
  - Mixture of susceptible and resistant cultivars (pitchounette): 6 orchards
- Period: season 2006 [May 30 - July 24]
- Pest: apple scab caused by ascomycete fungus: *Venturia inaequalis*
- Importance of climatic conditions:
  - *continuous measurements of Humidity and Temperature*
### Experimental Design

#### Figure 8: Plan schématique de la parcelle expérimentale AB04

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<th>mixte 1</th>
<th>réduc 1</th>
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- **Pure**: Culture pure de Melrouges
- **Mixte**: Culture en mélange
- **Réduc**: Culture en mélange avec réduction d’inoculum

1, 2 ou 3 : numéro du bloc

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*Haies composites non hôtes de la tavelure*
2 types of orchards

- melrouge tree (susceptible)
- pitchounette tree (resistant)

Mixed orchard

pure orchard
ascomycete fungus: *Venturia inaequalis*

artificial inoculation

scab symptoms
Venturia inaequalis Cycle

T=0
A data *driven* model and assumptions

1. Orchards were distant enough and separated by hedges
   Statistical independence of orchards
   but share the same dispersal mechanism and the same set of parameters

2. Space heterogeneity: void (paths), sensible and resistant cultivars affect spore diffusion:
   Introduction of a local displacement resistivity to dispersal
   ie “epidemiological distance” between locations

3. The location measurements only indicate the cardinal corner of the tree (~ 1 m²):
   Discretisation of space

4. Fungus dispersal took place only during favorable climatic conditions
   Usual time (in days) was inessential
   time is weighted by an infection severity index (sporulation conditions)
   Definition of a proper scab epidemiological time

5. Observation times: random and depend on climatic conditions and technician availability and should be considered as Markovian times (stopping times)
exemple of collected data (for a mixed orchard)

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Sequence of observations
Cumulative counts of infected leaves for the 9 orchards
Local space-time dynamics (nb of infected leaves)

Pure orchard 1

Mixed orchards (3, 5)

Nb of infected leaves in a tree quarter at 4 observation dates
Climatic conditions/epidemiological time

\[ \lambda(s) = 1, \quad \lambda(s) = 3, \ldots \]

Figure 17: Schéma de mise en relation des périodes à risques de contamination avec les conditions météorologiques
epidemiological space-time « coordinates »
or space-time transformation

1. Space: divided into cells with displacement resistivity $\rho$:
   $\rho(\text{void}=\text{reference})=1, \quad \rho(\text{susceptible}) = \alpha_{\text{Mel}}$ and $\rho(\text{resistant}) = \alpha_{\text{Pich}}$
   Pseudo distance between locations $X$ and $Y$:

   $D(X, Y) = \|Y - X\| \int_0^1 \rho(X + t(Y - X)) dt \approx \sum_{\text{cells } C_j : C_j \cap [X, Y] \neq \emptyset} \rho(C_j) \|C_j \cap [X, Y]\|$

2. Time: only at risk periods weighted by a severity coefficient (ecophysiology behavior of *Venturia inaequalis*) were counted:

   $\tau(s, t) = \int_s^t \lambda(u) du \approx \sum_{s < j \leq t} \lambda(j)$
Displacement resistivity and epidemiological contiguity

Example:
Euclidean distance $D_{euc}(X,Y) = 32.73483$  
Epidemiological « distance » $D_{epi}(X,Y) = 73.61998$

\[
p(x) = 1 \quad \text{and} \quad p(x) = 0.5
\]
Climatic conditions/epidemiological time

\[ \lambda(s) = 1, \quad \lambda(s) = 3, \ldots \]

**Figure 17**: Schéma de mise en relation des périodes à risques de contamination avec les conditions météorologiques
Natural modeling approach

- Multitype branching process

If $N(\tau_j) = (N(\tau_j, C_k); k=1,\ldots,M)$: counts of infected leaves in all cells $C_k$ (quarters of susceptible trees) observed at time $\tau_j$

- Model the infinitesimal generator of this Markovian process to take account of distances, climate, (easy task)
- Use Kolmogorov Equations and branching properties to set the system of diff. Eq for the set of conditional generating functions (or equivalently a system of linear PDE in this case)
- Solve the system ... (this is almost possible by approximation)
- Use inversion formula (or approximation) to recover the corresponding probability functions
- Use maximum likelihood techniques (intractable iteration procedure)

Not to do
A more sensible statistical model

Assumptions on dispersal and dynamics
- Additive and independent effect of infected leaves
- Markovian temporal behavior
- Multiplicative effect of proper time
- Exponential decrease of spore dispersal wrt epidemiological “metric”

Let \( N(\tau_j) = (N(\tau_j, C_k); k=1,\ldots,M) \) denote the counts of infected leaves in all cells \( C_k \) (quarters of susceptible trees) observed at time \( \tau_j \)

Likelihood (for one orchard)

\[
\prod_{\text{time } \tau_j} \prod_{\text{susceptible cells } C_k} \exp \left( -\lambda(\tau_j, C_k, N(\tau_{j-1})) \right) \lambda^{N(\tau_j, C_k)} \left( \tau_j, C_k, N(\tau_{j-1}) \right) / N(\tau_j, C_k)!
\]

\[
\lambda(\tau_j, C_k, N(\tau_{j-1}) \mid \theta) = \exp^{\alpha_{\text{base}}} + \sum_{p : N(\tau_{j-1}, C_p) \neq 0} \exp^{\alpha_{\text{Leaf}} + \alpha_{\text{Dist}} D(C_k, C_p \mid \theta) + \alpha_{\text{Time}} (\tau_j - \tau_{j-1})}
\]

where \( \theta = (\alpha_{\text{base}}, \alpha_{\text{Leaf}}, \alpha_{\text{Dist}}, \alpha_{\text{Time}}, \alpha_{\text{Mel}}, \alpha_{\text{Pich}}) \)

and \( D(C_k, C_p \mid \theta) = D_{\text{vo d}}(C_k, C_p) + \alpha_{\text{Mel}} D_{\text{Mel}}(C_k, C_p) + \alpha_{\text{Pich}} D_{\text{Pich}}(C_k, C_p) \)
## Results - Interpretation

<table>
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<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>Interpretation</th>
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<td>8.4807e-02</td>
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<tr>
<td>$\alpha_{\text{Dist}}$</td>
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<td>$\alpha_{\text{Time}}$</td>
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<td>$\alpha_{\text{Mel}}$</td>
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<td>$\alpha_{\text{Pitch}}$</td>
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<td>6.3916e-17</td>
<td>pitchounette resistivity</td>
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### Effect quantification

**Completely random (base) contribution**

$$\exp(\alpha_{\text{base}}) = 0.0586 \text{ infected leaf/cell}$$

**Multiplicative climat effect**: for a day at risk with severity of grade 2

$$\exp(\alpha_{\text{Time}} \times 2) = 1.412583$$
Results - interpretation

Contribution of a single infected leaf

Local contribution to its propre site (ie Distance = 0)
exp(\(\alpha_{\text{Leaf}}\)) = 0.2313092 : relative important contribution

Contribution of a single infected leaf to a site distant by 1-epidemiological distance during a day with severity 3
exp(\(\alpha_{\text{Leaf}} + \alpha_{\text{Dist}} \times \text{Dist} + \alpha_{\text{Time}} \times 3\)) = 3.036348e-05 : negligible contribution

Note however that « distances » within susceptible regions are also very low
1m(Euclidean or void) = 1m (resistant zone) = 0.0426m(susceptible zone)
Agronomic interest for mixed orchards